Fuzzy Model Based Predictive Control of Nonlinear Systems

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Abstract. In this paper, the generalized predictive control with the fuzzy model is developed. GPC has been developed to control linear time invariant plants. So, for controlling the nonlinear process, fuzzy model is used as the base model of the predictor. The fuzzy model predictive control is demonstrated by application to the problem of balancing and swing-up of an inverted pendulum on a cart.

Keywords. Predictive Control, Generalized Predictive Control, Fuzzy Logic, Model Based Predictive Control, Pendulum Reversed.

1 Introduction

Model-based predictive control (*MBPC*) is an advanced control strategy that has been widely applied in industry [1][2].

The generalized predictive control (*GPC*) of Clarke *et al.* [4][5] is one of the most popular *MBPC* algorithms. *GPC* is characterized by the prediction of a process model output over a prediction horizon, the evaluation of the difference between the desired and the predicted output for determining a sequence of controller outputs, and the optimization of an objective function over the prediction horizon including this difference and the control output. The optimization is repeated at every sample time with the new available process data, and at each step only the first control action in the calculated sequence is applied to the process (receding horizon principle).

The prediction is based explicitly on the model of the process to be controlled and it is presented as the first step in the predictive control. The predictive control based on the linear model works well for linear processes. However, the disadvantage of the predictive control with linear model is that it represents only the linear model of the process. Therefore, for nonlinear processes the predictive control would be unsatisfactory, then linearized model for approximating nonlinear process is required.

Recently, neural network and fuzzy logic theories have attracted considerable attentions to control nonlinear processes. Since, the predictive control requires the parametric form of the nonlinear system, the specific model in fuzzy logic is used based on Takagi-Sugeno's fuzzy model whose structure has a linear regression form

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in the consequence rule [7]. So, the name of this predictive control is called fuzzy model based predictive control [8][9][10].

In this work, we propose to apply the *GPC* algorithm to the class of nonlinear systems using fuzzy dynamic models. In this work we follow the steps to build first the fuzzy model of the system, show its validation, and then apply the predictive control to the obtained fuzzy model.

The paper is organized as follows : The generalized predictive control is presented in Section 2. Fuzzy model is presented in Section 3.1. In Section 3.2, we consider the prediction with fuzzy model. In Section 4, the design methodology is illustrated via a detailed example, namely the balancing and swing-up of an inverted pendulum on a car. Concluding remarks are collected in Section 5.

2 Generalized Predictive Control

Consider a dynamical system with input signal $u(t)$ and the output signal $y(t)$. Suppose that the signals are sampled in discrete time and that the sampled values are related trough the linear difference equation.

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})v(t)/\Delta \tag{1}$$

where $u(t)$, $y(t)$ and $v(t)$ are the input signal, output signal and disturbance process, respectively, at time t, $A(q^{-1})$, $B(q^{-1})$ and $C(q^{-1})$ are polynomials in the unit delay operator q^{-1}. The goal of the Δ operator ($\Delta = 1 - q^{-1}$) is to ensure integral action of the controller.

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A}$$
$$B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_{n_B} q^{-n_B}$$

The *GPC* algorithm consists in applying a control sequence that minimizes a multistage cost function defined as follows.

$$J_{GPC} = \sum_{j=N_1}^{N_2} [\hat{y}(t+j) - w(t+j)]^2 + \lambda \sum_{j=1}^{N_u} [\Delta u(t+j-1)]^2 \tag{2}$$

subject to $\Delta u(t+j-1) = 0$ for $j > N_u$. In this expression $\hat{y}(t+j)$ is the predicted output at time $t+j$ based on the available input/output data at time t ; $w(t+j)$ describes the future reference trajectory, N_1 is the minimum prediction horizon, $N_2 \geq N_1$ is called the maximum prediction horizon, $N_u \leq N_2$ is the control horizon and $\lambda \geq 0$ is a parameter which weights the relative importance of control effort.

Minimization of the above cost with respect to future control increments $\Delta u(t+j)$ $(j = 0, 1, \ldots, N_u - 1)$, together with the receding horizon, leads to

$$\Delta u(t) = [1 \quad 0 \quad . \quad . \quad . \quad 0] \left(G^T G + \lambda I\right)^{-1} G^T (W - F)$$

where the matrix G is of the form [4]

$$G = \begin{bmatrix} g_{N_1-1} & \cdot & g_0 & \cdot & \cdot & 0 \\ g_{N_1} & \cdot & \cdot & g_0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ g_{N_2-1} & \cdot & \cdot & \cdot & \cdot & g_{N_2-N_u} \end{bmatrix}$$

$$W = [w(t+N_1) \quad w(t+N_1+1) \quad . \quad . \quad . \quad w(t+N_2)]^{\mathrm{T}}$$

$$F = [f(t+N_1) \quad f(t+N_1+1) \quad . \quad . \quad . \quad f(t+N_2)]^{\mathrm{T}}$$

and I is the unit matrix. The elements of the matrix G are the step response values, and F is the free response of the open-loop system $B(q^{-1}) / A(q^{-1})$.

3 Fuzzy Model Based Predictive Control

3.1 Fuzzy Model

Fuzzy modeling for nonlinear systems is becoming more popular among control practitioners. The simplicity of the systems described by this technique gives to the practitioner a structured description of the system improving its capacity to analyze the behaviour of a given plant.

In the proposed design procedure, we represent a given nonlinear plant by the so-called Takagi-Sugeno fuzzy model [7]. This fuzzy modeling method is simple and can be very efficient to solve the problem of the parametric form. The system dynamics is captured by a set of fuzzy rules which characterize local relations in the state space. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy rule by a linear system model.

The Takagi-Sugeno (*TS*) fuzzy system is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of a system.

The fuzzy system is of the following form:

$$\begin{aligned} &Rule\ i: IF\ x_1(k)\ is\ M_{i1} \ldots and\ x_n(k)\ is\ M_{in} \\ &THEN\ x\ (k+1) = A_i x(k) + B_i u(k) \end{aligned} \tag{3}$$

where

$$\begin{aligned} x^T(k) &= [x_1(k), x_2(k), \ldots, x_n(k)] \\ u^T(k) &= [u_1(k), u_2(k), \ldots, u_m(k)] \end{aligned} \tag{4}$$

i=1,2,…,r and r is the number of IF-THEN rules. M_{ij} are fuzzy sets, and $x(k+1) = A_i x(k)+B_i u(k)$ is the output from the i[th] IF-THEN rule. Given a pair of $(x(k),u(k))$, the final output of the fuzzy system is inferred as follows :

$$x(k+1)=\frac{\sum_{i=1}^{r}\omega_i(k)\{A_i x(k)+B_i u(k)\}}{\sum_{i=1}^{r}\omega_i(k)} \tag{5}$$

r is the number of fuzzy rules and ω_i the firing strength of the i[th] fuzzy rule.

$$\omega_i(k)=\prod_{j=1}^{n}M_{ij}(x_j(k)) \tag{6.1}$$

Or

$$\omega_i(k)=\min\{M_{ij}(x_j(k))\} \tag{6.2}$$

$M_{ij}(x_j(k))$ is the grade of membership of $x_j(k)$ in M_{ij}.
Where it is assumed that

$$\sum_{i=1}^{r}\omega_i(k)>0 \qquad i=1,2,\ldots,r$$
$$w_i(k)\geq 0$$

The degree of fulfillment of the i[th] fuzzy rule is defined as :

$$\lambda_i(k)=\frac{\omega_i(k)}{\sum_{i=1}^{r}\omega_i(k)} \tag{7}$$

For all k, each linear component $A_i x(k)+B_i u(k)$ is called a subsystem.

3.2 Prediction with Fuzzy Model

The main idea of the fuzzy predictive control by using the *TS* fuzzy model is to compute the control action $u_i^{\bullet}(k)$ for each rule using *GPC* technique based on the considered linear sub-model. The control action computed for each rule minimizes the associated quadratic criterion. The global control action is the convex combination of the control actions computed for each fuzzy rule. It is given as :

$$u^{\bullet}(k)=\sum_{i=1}^{r}\lambda_i(k)\left(u_i^{\bullet}(k)+\alpha_i\right) \tag{8}$$

where $u_i^*(k)$ is the control action of the i[th] fuzzy rule, $\lambda_i(k)$ the degree of fulfillment of the i[th] fuzzy rule, α_i is a biasing parameter and $u^*(k)$ the global control action.

The first step of the algorithm is to build the fuzzy model given by equation Eq. (3). To find the parameters of the linear model, *Fuzzy Least Squares Method* is used, then, for each rule, one computes the optimal j-step ahead predictors, and then calculates the incremental control action for each rule. Only the first control element $u_i^*(k)$ of the sequence is applied to the process. The global control action is calculated by equation Eq. (8). Note that, the parameters used for the *GPC* essentially N_2, N_u and λ are not anymore fixed by the user but are the optimal parameters found by using a genetic algorithm [12].

Fig. (1) shows the structure of the predictive control based on the fuzzy model.

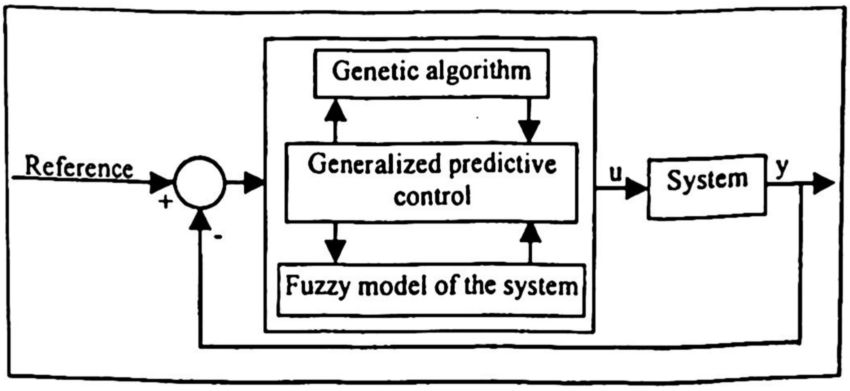


Fig. 1. Predictive control based on the fuzzy model

4 Application: Inverted Pendulum on a Cart

To illustrate the proposed approach, consider the problem of balancing and swing-up of an inverted pendulum on a cart, Fig. (2). The idea is to keep a pole vertically balanced. The top is attached at the bottom by a movable base. If the pole falls to the right or left, the base moves in the same direction to compensation.

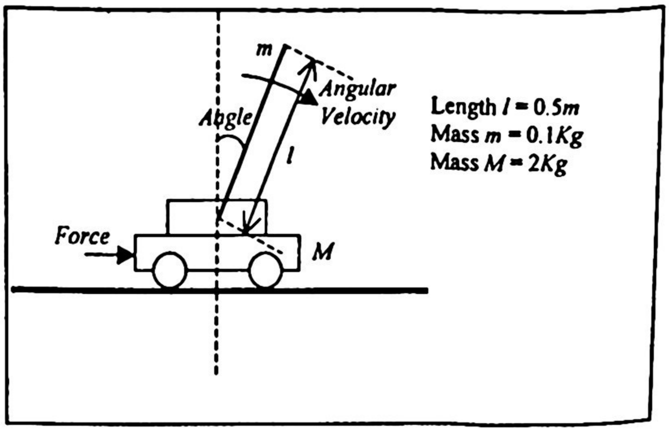


Fig. 2. Reversed pendulum

The equations of motion for the pendulum are, Eq. (9).

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{g\sin(x_1) - a\cos(x_1)u - am\,{}^{l}\!/_{2}\,x_2^2\sin(x_1)\cos(x_1)}{{}^{4l}\!/_{6} - am\,{}^{l}\!/_{2}\cos^2(x_1)} \end{cases} \tag{9}$$

where x_1 denotes the angle (in radians) of the pendulum from the vertical, and x_2 is the angular velocity. $g=10m/s^2$ is the gravity constant, m is the mass of the pendulum, M is the mass of the cart, l is the length of the pendulum, and u is the forced applied to the cart (in Newton), $a={}^{1}\!/_{M+m}$. We choose $M=2.0kg$, $m=0.1kg$, $l=0.5m$ in the simulations.

The fuzzy model is described by 4-fuzzy rules with 2 inputs $(x_1(k-1), x_2(k-1))$ and one output $(x(k))$ to get the following rules.

$$\begin{array}{c} Rule\ i : IF\, x_1(k-1)\ is\ M_{i1}\ and\, x_2(k-1)\ is\ M_{i2} \\ THEN\ x\ (k) = A_i x(k-1) + B_i u(k-1) \end{array} \tag{10}$$

where

$$x^T(k) = [x_1(k), x_2(k)]$$

Fig. (3) shows the membership function of the 2 inputs.

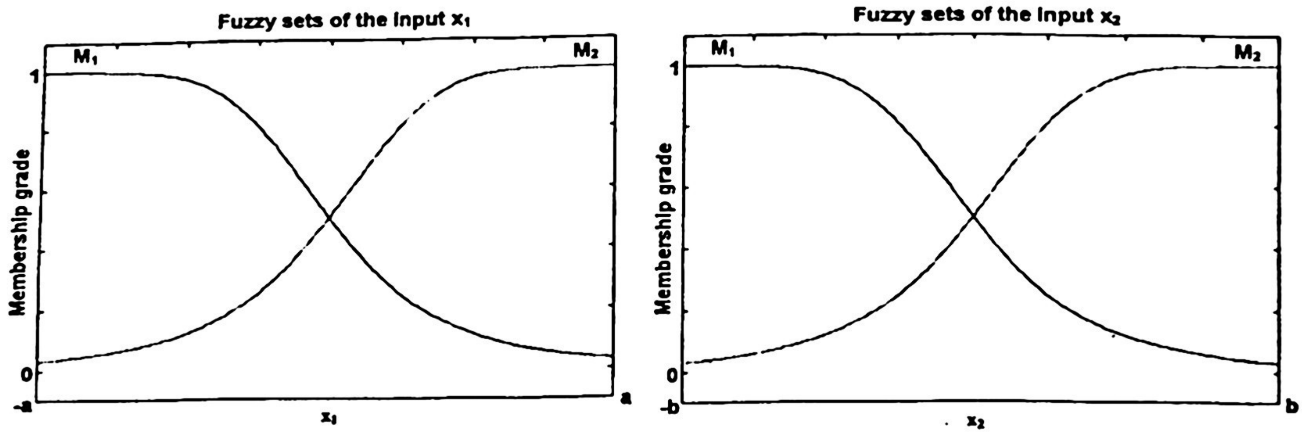
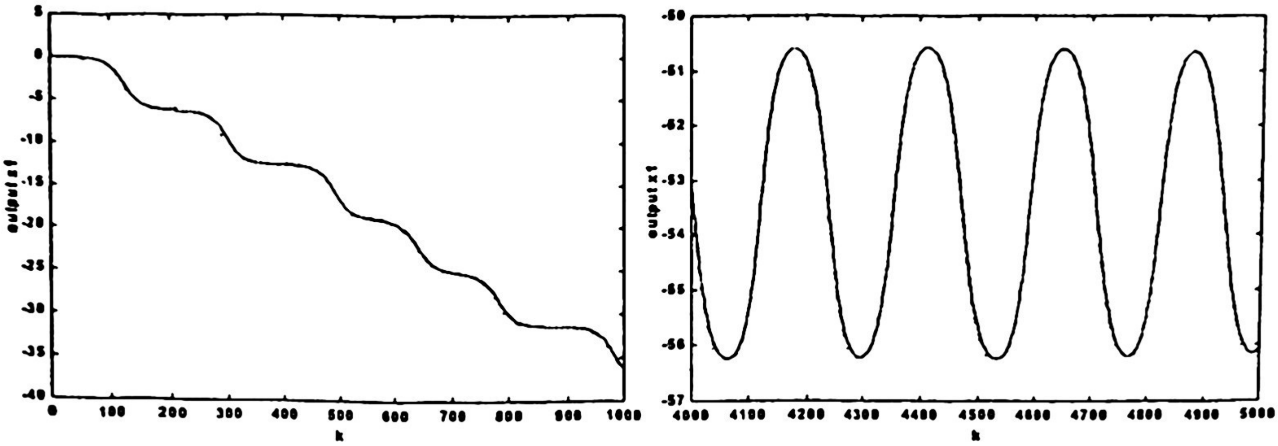


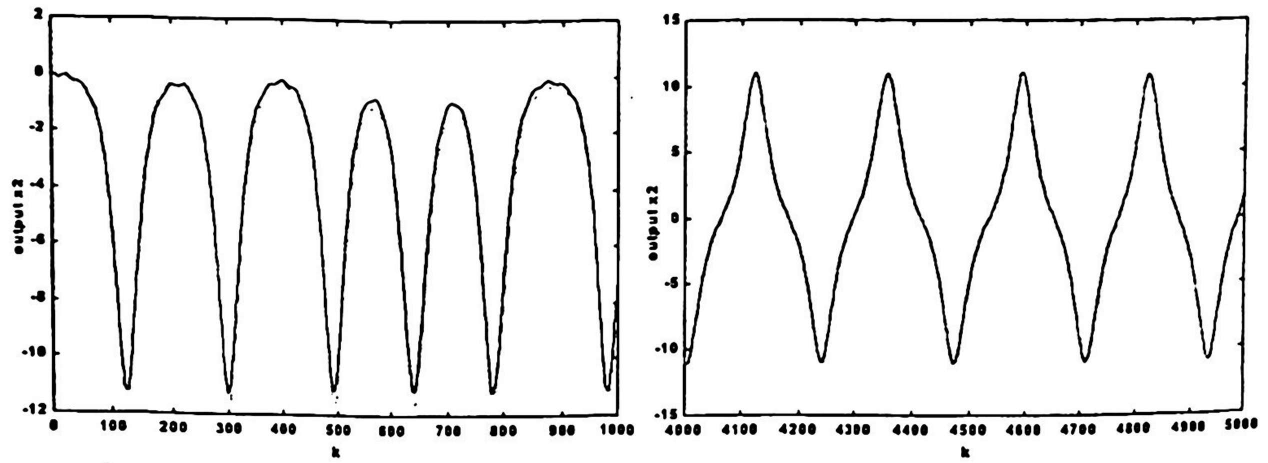
Fig. 3. Membership functions of the Example

Fig. (4) and Fig. (5) depict the curves of the nonlinear system and that of the fuzzy model. In Fig. (4), the solid line indicates the output x_1 of the nonlinear system and the dotted line shows the output x_1 of fuzzy model system, the result of identification is achieved for 5000 iterations, Fig. (4.a) shows answers in the interval $[0,1000]$ and Fig. (4.b) shows answers in the interval $[4000,5000]$. And also for the same thing for Fig. (5). It is clear that the model behave almost like the given system.



(a) (b)

Fig. 4. The output x_1 of the nonlinear system (line) and the output x_1 of fuzzy model system (dotted line)



(a) (b)

Fig. 5. The output x_2 of the nonlinear system (line) and the output x_2 of fuzzy model system (dotted line)

Now, we design a predictive controller based on this *TS* fuzzy model. The control objective is to balance the inverted pendulum for $x_1 \in (-a,a)$ and $x_2 \in (-b,b)$.

Simulations indicate (Fig. (6)) the control law can balance the pendulum for $a=\pi/4$ and initial conditions $x1=10°, 20°, 30°, 35°, 40°, 45°, \ldots$ and $b=2$. We remark that given the nonlinear plant Eq. (9) predictive control based on fuzzy model can be designed to balance the pendulum.

5 Conclusion

There are several approaches to control of a nonlinear system. In this paper, we have used fuzzy logic method. The fuzzy model based predictive control is developed and the developed predictor is based on the Takagi and Sugeno's fuzzy model. Predictive

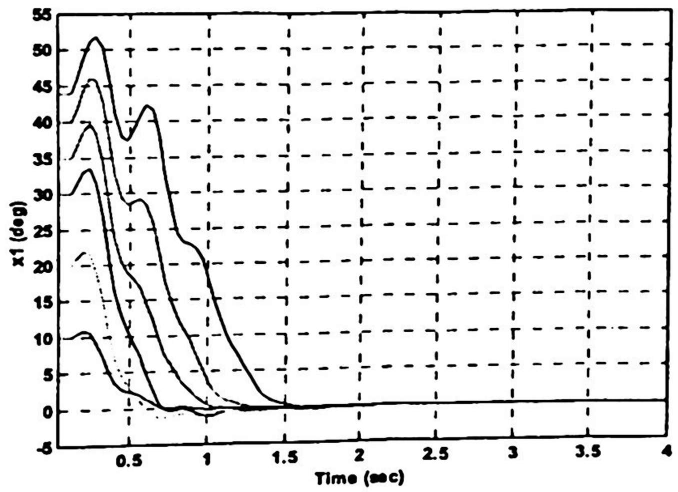


Fig. 6. Angle response

controllers for each of the sub-models are computed using classical techniques of the *GPC* algorithm and a global control action is subsequently computed as a combination of the controls obtained for each rule, using a fuzzy controller model similar to the fuzzy model of the process. A typical genetic algorithm is combined with a *GPC* algorithm to optimize the design parameters (N_2, N_u and λ) online. The designed methodology is illustrated by application to the problem of balancing and swing-up of an inverted pendulum on a cart.

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